JOURNAL OF MATERIALS SCIENCE 12 (1977) 1165–1175

# Crack growth in hybrid fibrous composites

I. R. McCOLL, J. G. MORLEY
*Wolfson Institute of Interfacial Technology, University of Nottingham, UK*

A preliminary analysis is made of the energetics of transverse crack growth in a brittle elastic matrix bridged by elastic fibres frictionally bonded to the matrix. Studies made of the stability of a crack of finite length in a brittle polymeric material reinforced with steel wires are found to be in reasonable agreement with the predictions of the theory. It is proposed that the stability of transverse cracks in a very brittle matrix could be increased substantially by the inclusion of a second fibre component designed specifically to increase the work of fracture of the matrix. This has been shown to be possible using a very small volume fraction of glass fibres as a matrix toughening component and it has also been observed that stable transverse matrix crack growth can be achieved with composite systems of this type. This principle might have applications in the design of hybrid composites utilizing either a brittle polymeric or ceramic matrix.

## 1. Introduction

In the case of a unidirectionally aligned fibrous composite, loaded in the direction of fibre alignment, multiple transverse cracking of the matrix is observed when its failing strain is less than that of the fibres. The factors controlling multiple cracking have been analysed for the case of parallel cracks extending across the full width of the material (see for example [1, 2]). It has been observed by Cooper and Sillwood [1], that the initiation of multiple matrix fracture can be suppressed if the fibres are sufficiently small. Multiple matrix fracture then occurs when an increased tensile strain is applied to the composite structure.

For some types of applications it is of technological importance to suppress matrix cracking in fibrous composites. For example, cracks can allow liquids or gases to escape through the material if it is used as some sort of container. Also matrix cracking increases the availability of the fibres and the fibre/matrix interface to environmental attack which can affect adversely the mechanical properties of the composite system.

On the other hand, multiple matrix cracking can provide a measure of strain-hardening and this is used to advantage, for example, in the design of glass fibre-reinforced plaster [3, 4]. Materials of this type do not fracture completely when fracture of the matrix first occurs because the crack-bridging fibres are capable of supporting the applied tensile load. At the crack faces the local tensile strain is assumed to correspond to the strain carried by the crack-bridging fibres, since the matrix cracks are considered to be, in effect, infinitely long. On each side of the crack, stress transfer between the fibres and matrix takes place, the numerical value being governed by the strength in shear of the interface and the absolute size of the fibres. Thus the tensile strain carried by the crack-bridging fibre falls with increasing distance from the crack faces. Differential movement between fibres and matrix therefore occurs and, since in most cases the interfacial interaction will be frictional, energy is absorbed through frictional losses. As the tensile load is increased, further progressive matrix cracking occurs, as less severe flaws in the matrix become operative, and the general tensile strain carried by the composite system increases. Energy is absorbed during this process. Hence the tensile load extension curve has characteristics similar to those of a strain-hardening metal but the overall failing strain of the composite is very small (usually in the region of 1%) being limited by the failing strain of the fibres.

As the fibre size is decreased, the volume of matrix on each side of a parallel crack which has relaxed elastically is reduced. If this is sufficiently small the amount of strain energy released may not be able to supply the energy required to rupture the matrix at its normal breaking strain. Matrix fracture then occurs at an enhanced strain value. This condition has been analysed by Aveston *et al.* [5], who also point out that it has some concomitent disadvantages.

First, the overall failure strain of the composite and the additional load it can support after the initiation of matrix fracture, are both reduced so that the composite system with thinner fibres may behave in an apparently more brittle fashion. Second, when the fibres are small, very localized high strain values in the composite can cause rupture of the reinforcing fibres. Such strains may be produced, for example, by localized flexing of a reinforced plate or in the vicinity of the tip of a notch when this is large compared with the fibre diameters and stress transfer lengths.

A third disadvantage, arising from the use of small fibres, is that the use of some candidate fibres is prevented on economic grounds. For example, it does not seem practicable to increase the tensile failing strain of a cement matrix through the use of very thin steel wires because the economics of the building industry preclude the use of a sufficient volume fraction of sufficiently small diameter steel wires [5]. However, the use of glass fibres to reinforce gypsum plaster has proved to be very attractive [3, 4]. In this system, the strain at which the plaster matrix first cracks can be higher than that of the unreinforced matrix and, in addition, multiple fracture of the matrix also occurs before the failing strain of the composite is reached.

Recently, studies have been made of the mechanics of matrix fracture by transverse cracking in composite systems containing non-fracturing reinforcing elements. The cracks are bridged by the reinforcing members which carry tensile loads across the crack faces, reduce the amount of strain energy released by the crack and also absorb energy as a consequence of frictional interactions taking place through differential movements between the fracturing and non-fracturing part of the composite structure [6, 7]. A theoretical analysis has been developed by use of which estimates can be made of the rate of release and absorbtion of energy during crack extension. Experimental observations of the behaviour of metal sheets and polymeric sheets, containing a crack bridged by non-fracturing reinforcing elements, were found to be in agreement with the general predictions of the theory.

In this paper the analysis previously developed is applied to the mechanics of the growth of a transverse matrix crack in conventional unidirectional fibrous composites. It is applicable where the matrix is brittle and has a lower failing strain than the fibres. Also the fibres must be capable of supporting the total load applied to the composite system so that the crack-bridging fibres do not exceed their failing strain. Furthermore, stress concentrations at the crack tip are considered to be eliminated by interfacial debonding so as to prevent fibre failure in this region. The analysis deals with cracks of finite length and predicts that, when the matrix is brittle having a very low work of fracture, the unstable growth of a matrix crack would be expected at fairly low overall strain values. This is, of course, observed in the form of unstable multiple matrix cracking in brittle matrix composites discussed above. In addition, the analysis indicates the manner in which matrix crack growth would be expected to be suppressed by increasing the work of fracture of the matrix. This leads directly to a consideration of the use of specialized strengthening and toughening components in brittle matrix composites.

By increasing the work of fracture of the matrix, through the use of a specilized toughening component, it becomes possible to suppress cracking in a brittle matrix while still making use of relatively large strengthening and stiffening fibres as the primary reinforcing component. Such an approach may have technical and economic advantages. Preliminary studies of the behaviour of hybrid systems of this type have been carried out and are reported below. It has been shown to be possible to enhance the composite tensile strain at which a matrix crack starts to propagate and, in addition, to arrange for such a crack to be stable so that its extension is governed by the additional load applied to the composite system. Thus stable multiple matrix fracture at enhanced matrix strain values seems possible in brittle matrix composite systems.

The experimental samples used have utilized unidirectional steel wires (as the strengthening component) and a random planar array of glass fibres (as the toughening component) in a very brittle epoxy resin matrix. Both the toughening and reinforcing fibres have been present in very low concentrations (~ 1%). Although glass fibres have been used as the toughening component there would seem no reason why polymeric fibres should not be utilized for the same purpose since they are not called upon to stiffen the matrix.

## 2. Transverse matrix fracture in unidirectional fibrous composites

A theoretical analysis of the strain field around a transverse matrix crack of finite length in a unidirectional fibrous composite subjected to a general tensile strain in the direction of fibre alignment has been developed in detail elsewhere [6, 7]. This analysis is given in outline below.

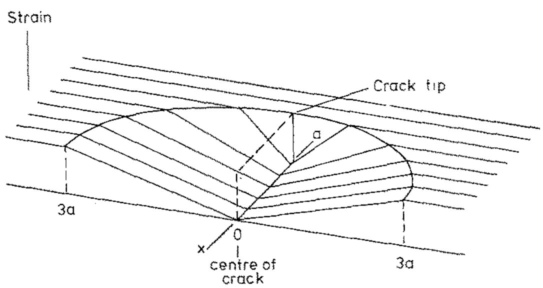


*Figure 1* Idealized strain distribution around a crack in an unreinforced sheet.

An elliptical zone around the crack in the unreinforced matrix is assumed to be partly relaxed. This is illustrated in Fig. 1. A linear change in tensile strain is assumed parallel to the direction of loading from the crack face to the extremities of the elliptical zone. Strains other than longitudinal tensile strains are neglected and the size of the elliptical zone is chosen so that, on the basis of this assumption, the amount of strain energy released by the unreinforced matrix corresponds to that obtained by a complete integration of the strain field around a crack in an elastic solid. It follows that the major axis of the ellipse is three times the crack length.

The continuous fibres bridge the crack orthogonally and stress is transferred between the matrix and the fibres through a constant shear strength interface. The strain distribution along one fibre and the surrounding matrix from the position of the crack to the edge of the ellipse is shown in Fig. 2. The maximum load, and hence the maximum fibre strain, occurs at the position of the crack. This is given by $\epsilon_\mu$ where $\epsilon_\mu$ is less than the failing strain of the fibre. Load is transferred from the fibre to the matrix in the vicinity of the crack (from 0 to $L_1$). The rates of change of fibre strain and matrix strain in the vicinity of the crack are shown by the slopes of the lines VQ and OQ in Fig. 2 and are given respectively by,

$$d\epsilon_f/dx = -2\tau/E_f r. \qquad (1)$$

and

$$d\epsilon_m/dx = 2V_f\tau/E_m V_m r + \epsilon_\beta/L_3. \qquad (2)$$

where $2r$ is the diameter of the fibre, $\tau$ is the frictional shear strength of the fibre/matrix interface. $E_f$ and $E_m$ are the Young's moduli of the fibre and matrix, respectively; $V_f$ and $V_m$ are the volume fractions of fibres and matrix, respectively; $\epsilon_\beta/L_3$ is the strain gradient along the section in the absence of the reinforcing elements; $\epsilon_\beta$ is the general tensile strain in the composite outside the elliptical zone; $L_3$ is the perpendicular distance from the plane of the crack to the edge of the elliptical zone and is given by, $L_3 = 3(a^2 - x^2)^{\frac{1}{2}}$,

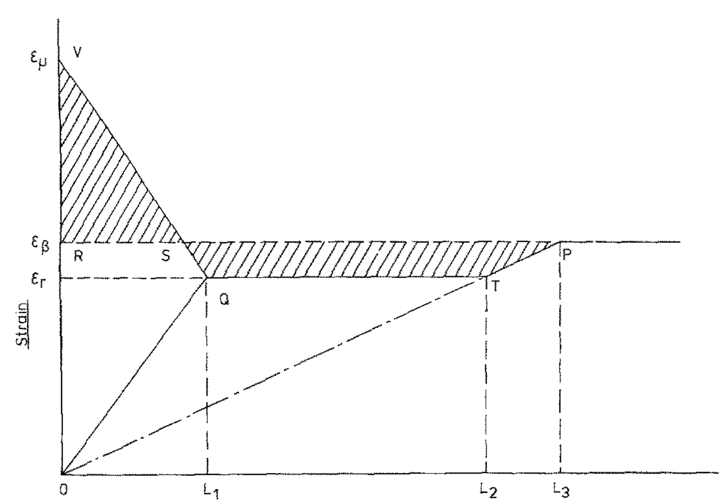


*Figure 2* Strain distribution along a section perpendicular to a matrix crack in a sheet reinforced with a parallel array of steel wires.

where $a$ is the half crack length and $x$ is the distance from the centre of the crack to the fibre considered.

At the position $L_1$ the tensile strains in fibre and matrix are the same and no further stress transfer takes place. The strain in both fibres and matrix is assumed to remain constant from $L_1$ to $L_2$. From the position $L_2$, which lies on the strain distribution for the unreinforced matrix, to the edge of the elliptical zone the strain distributions in both fibres and matrix are assumed to be the same and to correspond to that of the unreinforced matrix.

Since the perturbations in the strain field generated by the matrix crack are assumed to be confined to the elliptical zone defined above, the increase in the length of the fibre near the crack must be balanced by the reduction in its length resulting from the decrease in fibre tensile strain elsewhere. Hence the areas of the triangles shown shaded in Fig. 2 must be equal.

This boundary condition together with Equations 1 and 2 enables equations defining the strain distribution along any fibre and the surrounding matrix to be obtained. These are:

$$\left.\begin{aligned} \epsilon_r &= [\epsilon_\beta L_3/\{Q(P+\epsilon_\beta/L_3)^{-2}+L_3/\epsilon_\beta\}]^{\frac{1}{2}} \\ \epsilon_\mu &= L_1(P+Q+\epsilon_\beta/L_3) \\ L_1 &= \epsilon_r(P+\epsilon_\beta/L_3)^{-1} \\ L_2 &= L_3\epsilon_r/\epsilon_\beta \\ L_3 &= 3(a^2-x^2)^{\frac{1}{2}} \end{aligned}\right\} \quad (3)$$

where $P = 2V_f\tau/E_m V_m r$ and $Q = 2\tau/E_f r$. $\epsilon_r, \epsilon_\mu, \epsilon_\beta, L_1, L_2, L_3$ are defined in Fig. 2.

The half-crack opening at a distance $x$ from the centre of the crack is obtained from the difference in integrated strain between the reinforcing fibres and the matrix over the distance $OL_1$ and is given by,

$$\mu = \epsilon_\mu L_1/2. \quad (4)$$

The strain energy, $\delta WR_x$, released by a parallel-sided section of the composite of width $\delta x$ and unit thickness at a distance $x$ from the centre of the crack, may be obtained from Equation 3. Thus,

$$\begin{aligned} \delta WR_x = {} & [E_c\epsilon_\beta^2 L_3/2 - E_c\epsilon_\beta^2(L_3^3-L_2^3)/6L_3^2 \\ & -E_c\epsilon_r^2(L_2-L_1)/2 - V_m E_m \epsilon_r^2 L_1/6 \\ & -V_f E_f(\epsilon_\mu^2+\epsilon_\mu\epsilon_r+\epsilon_r^2)L_1/6]\,\delta x \end{aligned} \quad (5)$$

where $E_c = E_m V_m + E_f V_f$.

The total strain energy released by the presence of the matrix crack can be computed by numerically integrating Equation 5 over the whole of the elliptical zone. The rate of release of strain energy with increasing crack length can then be computed by numerical differentiation of the strain energy released at increasing crack lengths.

The work done against frictional forces during the growth of the crack can be computed in the following manner.. First, a fibre located at a distance $x$ from the mid-point of the crack is considered. During crack growth, relative displacements between the fibre and the matrix occur between the crack face and the position $L_1$ (Fig. 2), so that the frictional energy losses are confined to this region.

The relative displacement $dm_y$ at a distance $y$ from the crack face is given by the integrated strain difference between the fibre and matrix from the position $y$ to $L_1$. Now $\epsilon_\mu\ (1-y/L_1)$ is the strain difference between fibre and matrix at a distance $y$ from the crack face so that:

$$\begin{aligned} dm_y &= \int_y^{L_1} \epsilon_\mu(1-y/L_1)dy \\ &= \epsilon_\mu(L_1/2-y+y^2/2L_1) \end{aligned}$$

where $\epsilon_\mu$ and $L_1$ are given by Equations 3. The frictional force across a length dy of the fibre/matrix interface will be given by $2\pi r\tau dy$. Hence the frictional work done by the length $dy$ of the fibre at a distance $y$ from the crack face during the growth of the crack is given by,

$$2\pi r\tau dy dm_y = 2\pi r\tau dy \epsilon_\mu(L_1/2-y+y^2/2L_1).$$

The total work done by the fibre is obtained by integrating this expression from 0 to $L_1$ and is

$$\pi r\tau\epsilon_\mu L_1^2/3.$$

The number of fibres in a section of width $\delta x$ and unit thickness is $V_f\,\delta x/\pi r^2$. Hence the work done against frictional forces in this section of the quadrant of the ellipse is given by,

$$\delta WA_x = V_f\tau\epsilon_\mu L_1^2\delta x/3r.$$

Substituting for $\epsilon_\mu$ from Equation 3 gives

$$\delta WA_x = V_f\tau\delta x\{P+Q+\epsilon_\beta/L_3\}L_1^3/3r. \quad (6)$$

The total strain energy absorbed frictionally can be obtained by numerically integrating Equation 6 over the whole partially relaxed elliptical zone around the crack. The rate of absorption of energy by this mechanism with increaisng crack length can then be obtained by numerically differentiating the strain energy absorbed at different crack lengths.

In order that a transverse matrix crack should grow the rate of release of strain energy with increasing crack length must equal or exceed the rate of absorption. The work of fracture of the matrix has thus to be added to the energy absorption term as does also any energy absorbed by the chemical debonding of the fibre matrix interface.

## 3. Application of the analysis of transverse matrix fracture to the design of hybrid composite systems

The analysis outlined in Section 2 has been used to compute the fracture characteristics of a wide range of composite systems [9]. These studies have shown the rate of energy absorption by frictional effects is, in general, about one half of the rate of energy released as the matrix crack propagates. The situation is illustrated by the computed values of the rates of release and absorption of energy for specimen 5 (Tables I and II) which are plotted in Figure 6. Although the presence of the crack-bridging fibres can reduce the rate of release of strain energy by the matrix by several orders of magnitude compared with the unreinforced material, transverse crack growth in the matrix will still occur at low strain values if the intrinsic work of fracture of the matrix is very low. If the rate of release of strain energy during the growth of a matrix crack is reduced by, for example, increasing $V_f$ or $E_f$ or $\tau$, the frictional energy loss term will be correspondingly reduced. It is, therefore, desirable to be able to increase the intrinsic work of fracture of the matrix independently of the strain energy locking mechanism and frictional energy losses outlined in Section 2.

This situation is, of course, achieved if the matrix is a metal and the very considerable advantages in crack stability and damage tolerance of sheet metal reinforced with crack-bridging reinforcing members have been reported elsewhere [6, 7].

When the matrix is a brittle polymer, its work of fracture may be increased by the addition of a dispersed particulate rubber phase. A progressive reduction in the incidence of matrix cracking with increasing volume fraction of the rubber particles was reported by McGarry [8]. The system studied was a polyester resin reinforced by 60 wt % woven glass fabric. The rubber phase used was C.T.B.N. (a carboxyl terminated butadiene/acrylonitrile copolymer), which was added in various volume fractions. The composite system was subjected to 100 tensile cycles at 50% and 75% of its UTS and a dramatic reduction in the incidence of matrix cracking was observed with increasing volume fraction of the rubber particles.

As an alternative to rubber particles, it was considered that the work of fracture of the polymeric matrix might also be increased by a suitable additional fibre phase. It was necessary to ensure that this second phase would behave differently from the first and not function simply as an addition to the existing primary reinforcing phase.

A number of factors have to be taken into consideration in the design of the toughening fibre phase and some of these are not readily susceptible to analysis. Factors such as matrix splitting, the tearing out of fibres aligned at an acute angle to the fracture face and the influence of these factors on fibre debonding and fibre pull-out need to be considered.

The work absorbed by pulling out fibres which are aligned perpendicularly to the fracture face is more amenable to analysis, and estimates of the maximum amount of energy which can be absorbed in this way can be made. The magnitude of the displacement available is set by the width of the crack at any point as described in Equation 4. The maximum contribution to the matrix work of fracture by the toughening fibre phase will be obtained when the stress carried by the fibre, as the crack faces separate, increases as rapidly as possible to just less than the fibre UTS and then falls as little as possible as the fibre is extracted from the matrix. It is clear from these considerations that the maximum work of fracture will be obtained, in general, when the toughening fibre phase has a smaller diameter than the primary fibre phase. For example, if the toughening fibres had properties identical with those of the primary phase they would be experiencing the same strain distribution in the vicinity of the crack as those of the primary phase. Now the general tensile strain carried by a brittle matrix composite, when matrix fracture is initiated, can be two orders of magnitude less than the failing strain of the primary reinforcing fibres. It is clear, therefore, from Fig. 2 that at the onset of matrix fracture, large diameter crack-bridging matrix-toughening fibres would be carrying a much lower strain than their ultimate tensile strain and hence would be contributing very much less to the work of fracture of the matrix than would smaller diameter fibres of the same type. The rate of loading up the toughening fibres as the crack faces separate will depend on the Young's modulus of the fibres as well as their diameters, and the pull-out process will, of course, depend on the distribution of fibre flaws.

In order to develop the maximum contribution to the matrix work of fracture, the toughening fibre has to fracture at a point as far away from the fracture face as possible. This cannot, of course, exceed half the fibre critical length if pull-out is to occur. The maximum possible contribution of the toughening fibres to the work of fracture may be derived in the following manner. Firstly, it is assumed that the fibres are all oriented perpendicular to the fracture face. Secondly, they are assumed to be of length equal to the fibre critical length and to have their mid-points positioned on the plane of fracture. From the theory set out in Section 2 the work done per unit cross-section, $W_1$, in raising the strain carried by the fibres to $\epsilon_\mu$, at the point where they emerge from the fracture face, is given by,

$$W_1 = V_f \tau \epsilon_\mu L_1^2/3r$$

substituting $\epsilon_\mu = \sigma_f/E_f$ and $L_1 = l_c/2$ where $\sigma_f$ is the fibre UTS. and $l_c$ is the fibre critical length. ($l_c = r\sigma_f/\tau$), we have,

$$W_1 = V_f \frac{\tau}{3r} \frac{\sigma_f}{E_f} \left(\frac{l_c}{2}\right)^2$$

$$\text{and } W_1 = \frac{V_f}{12} \frac{\sigma_f^2}{E_f} l_c$$

From Equation 4, and again substituting $l_c$ for $L_1$ we have for the half crack opening, $\mu$, when the fibre stress at the crack face is $\sigma_f$

$$\mu = \frac{\sigma_f l_c}{4E_f}$$

Hence, the amount by which the crack faces must spearate in order to load a fibre of length $l_c$, equally disposed on each side of the crack, to its UTS is given by,

$$\sigma_f l_c/2E_f.$$

Now if the crack faces separate by a greater amount the fibres will be extracted from the matrix and work will be done against frictional forces. If this work is called $W_2$ we have

$$W_2 = V_f\sigma_f \left[z - \frac{\sigma_f l_c}{2E_f}\right] \left[1 - \frac{1}{l_c} \frac{(z - \sigma_f l_c)}{2E_f}\right]$$

where $z$ is the final separation of the crack faces. Rearranging

$$W_2 = V_f\sigma_f \left[z - \frac{z^2}{l_c} + \frac{z\sigma_f}{E_f} - \frac{\sigma_f}{2E_f} l_c - \frac{\sigma_f^2 l_c}{4E_f^2}\right]$$

and the total work done $= W_1 + W_2 = W_{tot}$ where

$$W_{tot} = V_f\sigma_f \left[z - \frac{z^2}{l_c} - \frac{5\sigma_f l_c}{12E_f} + \frac{z\sigma_f}{E_f} - \frac{\sigma_f^2 l_c}{4E_f^2}\right]$$

and

$$\frac{dW_{tot}}{dl_c} = V_f\sigma_f \left[\frac{z^2}{l_c^2} - \frac{5\sigma_f}{12E_f} - \frac{\sigma_f^2}{4E_f^2}\right]$$

hence, $W_{tot}$ has a maximum value when

$$l_c = \frac{z}{[(\sigma_f/4E_f)(5/3 + \sigma_f/E_f)]^{1/2}}. \qquad (7)$$

Hence, for maximum energy absorption,

$$l_c \simeq 2zE_f^{\frac{1}{2}}/\sigma_f^{\frac{1}{2}}$$

or,

$$r/\tau \simeq 2zE_f^{\frac{1}{2}}/\sigma_f^{\frac{3}{2}}.$$

Hence, in order to maximize the work done by these mechanisms the toughening fibre diameter must be reduced as the separation of the crack faces is reduced. Also the optimum toughening fibre diameter will depend approximately on the ratio between the fibre elastic modulus and its failure stress.

Equation 7 can be regarded only as a rough guide to the optimum diameter of the second toughening fibre phase because of the effect of other factors previously mentioned above and because the toughening fibres would exert a tractive force across the crack faces and influence the effect of the primary fibres. Taking all these considerations into account it was decided to investigate the behaviour of a random planar array of thin glass fibres as the secondary toughening phase in conjunction with much thicker unidirectional steel wires as the primary phase. It was felt that even the small separation of the crack faces expected in the matrix would be sufficient to cause fracture of the thin glass fibres and ensure energy absorption by matrix splitting and fibre pull-out. The use of a random two-dimensional array of fibres would also ensure that there would be some degree of matrix tearing and fibre pull out from fibres aligned at an acute angle to the plane of the crack. Also the contribution of the second phase fibres to the longitudinal stiffness of the composite, and hence the rate of release of strain energy, would be minimized.

## 4. Experimental arrangements

Experimental samples in the form of relatively thin flat plates were used. These contained a central crack of known dimensions and were loaded in tension in a direction perpendicular to that of the alignment of the crack. End plates were attached to the samples to facilitate their attachment to grips which were attached to pivoted couplings at their centre points. Four types of experimental samples were used: (a) plain resin specimens (Ciba Geigy MY 750 with 10% by weight of HY 951 hardener cured at room temperature for three days); (b) resin specimens reinforced with glass fibre, randomly aligned in the plane of the sheet; (c) resin specimens reinforced with continuous steel wires aligned in the loading direction; and (d) resin specimens reinforced with glass fibre and steel wires. The volume fractions of both wires and glass fibres were quite small (see Table I). The rate of straining was in all cases 0.2 mm $min^{-1}$ and the tensile strain in each specimen was taken as the machine cross-head movement divided by the nominal length of the specimen. The crack was formed by fixing a thin cardboard sheet 40 mm long and coated with release agent in the centre of the mould prior to casting the resin. Thus the crack was formed in the resin or glass fibre-reinforced resin matrix and was bridged by the steel wires. The radius of curvature of the tips of the crack was set by drilling two 1 mm diameter holes at 40 mm spacing.

The wire used was 18/8 hard drawn stainless steel, nominal diameter 0.6 mm, having a UTS of 1.6 GN $m^{-2}$. The surface condition of the wire was as-received except for a degreasing step. The wires were arranged as a single layer in a parallel array. The distance between centres was 6.35 mm, each specimen containing 24 wires. The glass fibre used was commercially available polyester surfacing tissue (Part No. LGT9 Spectra Chemicals, Haywards Heath, Sussex). It was available in the form of flat sheets nominally 1 mm thick with the fibres arranged randomly in the plane of the sheet.

Preliminary studies were made of the forces developed in pulling a single length of wire ~ 150 mm out of a block of resin. This was carried out in order to establish whether or not any significant amount of energy was absorbed in debonding the wire. The data obtained is shown in Fig. 3. Debonding occurred suddenly at a tensile load of about 150 N when a 35 mm length of wire was

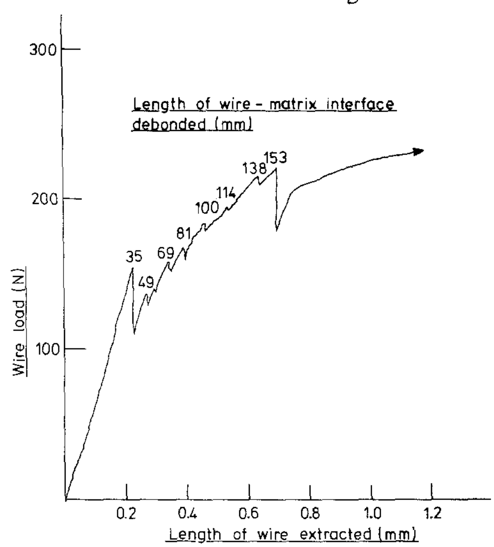


*Figure 3* Observed load–length withdrawn curve, for the extraction of a steel reinforcing wire from a resin block.

TABLE I Details of experimental specimens

| Specimen no. | Type | Sheet thickness (mm) | Sheet width (mm) | Gauge length (mm) | Volume fraction of glass fibre | Volume fraction of wire |
|---|---|---|---|---|---|---|
| (1) | Resin only | 3.8 ± 0.3* | 153 | 227 | 0 | 0 |
| (2) | Resin reinforced with glass fibre | 3.3 ± 0.5 | 152 | 237 | 0.0157 | 0 |
| (3) | Resin reinforced with stainless steel wire | 3.5 ± 0.4 | 155 | 236 | 0 | 0.0126 |
| (4) | Resin reinforced with glass fibre and | 3.5 ± 0.3 | 154 | 234 | 0.0146 | 0.0126 |
| (5) | stainless steel wire | 3.8 ± 0.3 | 155 | 234 | 0.0134 | 0.0115 |

*Non-uniform sheet thickness due to casting techniques employed.

TABLE II Observed and computed failure stresses

| Specimen no. | Observed failure initiation stress ($MN\,m^{-2}$) | Observed failure completion stress ($MN\,m^{-2}$) | Computed failure stress ($MN\,m^{-2}$) | |
|---|---|---|---|---|
| | | | ($\tau = 0.75\,MN\,m^{-2}$) | ($\tau = 1.50\,MN\,m^{-2}$) |
| 3. | 3.18 | 5.06 | 3.4 | – |
| 4. | 7.60 | 10.20 | 7.9 | 9.0 |
| 5. | 6.96 | 10.02 | 7.6 | 8.6 |

observed to have debonded. As the load was increased the debonded length increased progressively and fairly regularly until the whole length of wire was debonded at a load of about 220 N. The length of wire corresponded to about half the length of the experimental specimens and it was concluded that, in this particular system, chemical debonding was not a factor of major significance. The pull-out load developed by the residual frictional interaction corresponded to about one half of the UTS of the wire and the frictional shear strength of the wire resin interface was about $0.75\,MN\,m^{-2}$.

## 5. Experimental observations on flat plate specimens – comparison with basic theory

Both the unreinforced resin specimens and the glass fibre reinforced resin specimens behaved as conventional brittle solids, catastrophic crack growth occurring at a critical load and strain value. Experimental data for such specimens are shown in Fig. 4. Nominal work of fracture values $G_c$ were obtained using the classical Griffith relationship:

$$G_c = \frac{\pi a \sigma^2}{E} \tag{7}$$

From Equation 7 values of $G_c$ calculated for the unreinforced resin and the glass fibre-reinforced resin were 0.021 and $0.26\,kJ\,m^{-2}$ respectively.

The stress to cause unstable crack growth of the matrix in the wire-reinforced resin specimen was obtained from the experimental load extension data shown in Fig. 4. In this case the specimen did not fracture completely since the wires continued to carry a load across the crack faces. Since the mechanical properties of this system were known it was possible, using the theory given in Section 2, to calculate the stress at which a 40 mm long crack in an infinite sheet would be expected to propagate. This was found to be in tolerable agreement with the observed value for the experimental specimen used (see Table II). In the case of this specimen, the central crack first propagated to one edge of the sheet and subsequently, at a higher stress value, to the other edge of the sheet.

Similar calculations were performed for the hybrid system consisting of resin reinforced with steel wires plus glass fibre. The value of $G_c$ for the matrix was taken to be the same as that calculated at the onset of unstable crack growth for the glass fibre-reinforced resin system. The theory predicts that unstable matrix crack growth will occur at a

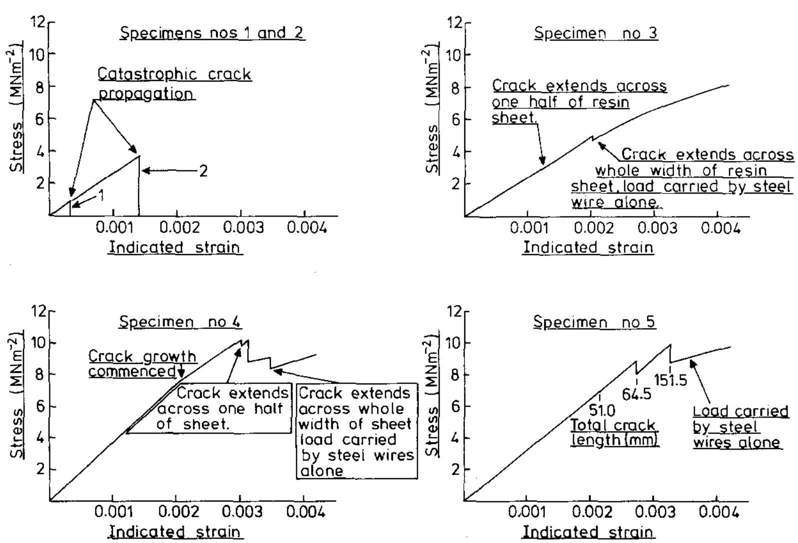


*Figure 4* **Observed stress–indicated strain diagrams for various unreinforced and reinforced sheets described in Table I**

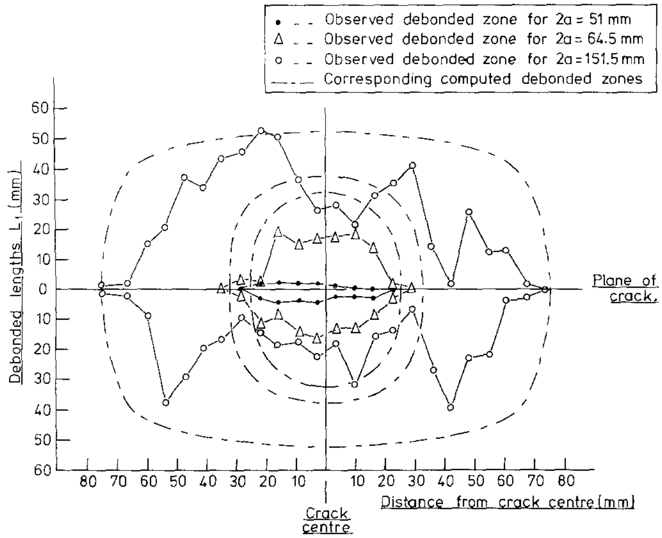


*Figure 5* Observed and computed values of the debonded lengths ($L_1$) of the wire–matrix interface at various crack lengths.

critical stress level but this was not observed (Fig. 4). Matrix crack growth began at a rather lower stress than that predicted by the theory but remained stable. Crack extension took place progressively as increasing tensile strains were applied to the material.

This can be explained as follows. The theory given in Section 2 deals with a fully developed stress field in which the crack-bridging reinforcing members are carrying a maximum limiting strain set by the bulk tensile strain carried by the composite system as a whole. This implies that stress transfer occurs between the crack face and the position $L_1$ (Fig. 2). This is not the case when load is first applied to the system but progressive debonding of the wire can be observed and the position of the limit of interfacial debonding at various crack lengths and applied loads is shown in Fig. 5. Also shown in Fig. 5 is the debonded limit which would be expected for cracks of various lengths according to the theory set out in Section 2. The theory, of course, refers to a crack in a sheet of infinite size and will thus become suspect as the crack length becomes comparable with the width of the sheet. It will be seen that the observed limit of experimental debonding is less than that predicted by the theory although it is comparable with it. It follows that, under experimental conditions the crack-bridging fibres are not carrying their maximum limiting strains and frictional energy losses are not occurring over the maximum length of interface predicted by the theory. Hence, at a particular crack length, the amount of strain energy released under experimental conditions is greater and the amount of strain energy absorbed is less than that predicted by the theory. Thus matrix crack growth might be expected to occur at a rather lower level than that predicted by the theory.

It is interesting to compare the rate of release and rate of absorption of energy at different crack lengths. Values computed on the basis of the theory are shown in Fig. 6. It will be seen that for

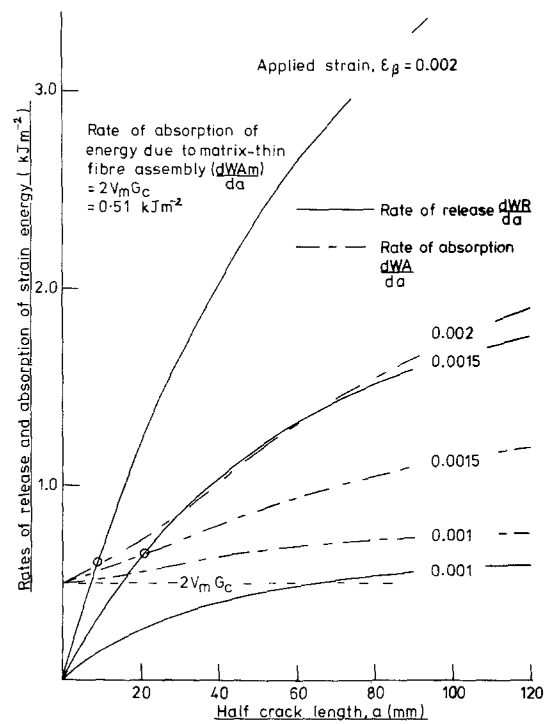


*Figure 6* Rates of release and absorption of strain energy computed using data appropriate to specimen no. 5.

small departures from the critical crack growth condition the difference between the rate of energy released and rate of energy absorbed will be very small. This observation together with the knowledge that initial crack extension is occurring before the strain energy locking mechanism and frictional energy absorption mechanism is fully developed can account for the stable progressive increase in matrix crack length which is observed. However, the calculated stress values for matrix crack extension are similar to those observed experimentally (Table II). The theory indicates that the crack stability should be relatively insensitive to changes in $\tau$ for the large primary load bearing fibres. In Table II critical stress values calculated on the basis of a factor of two increase in values of $\tau$ are shown. The critical stress increase is only of the order of 10%.

## 6. Discussion

The simplified analysis set out in Section 2 is capable of predicting strain distributions around a transverse crack in a frictionally bonded fibre matrix system as well as strain energy release rates and rates of absorption of energy by frictional effects during crack extension. The analysis shows that the frictional energy loss term, although very important, is generally about 50% of the corresponding rate of strain energy release. Hence unstable transverse matrix crack growth is still possible if the matrix has a very low work of fracture.

Stable cracks have been observed in metal matrix systems reinforced with crack-bridging non-fracturing reinforcing members (for which the present analysis is also applicable) [7]. As discussed above a reduction in the tendency for matrix cracking has also been observed elsewhere [8], when a glass fibre reinforced polymeric matrix is toughened by the inclusion of rubber particles in the matrix.

The preliminary observations reported here indicate the feasibility of using a second fibre phase to toughen a brittle matrix and enhance the tensile strain at which a pre-existing flaw will propagate. For the particular system studied the presence of a total fibre volume fraction of about 3% increased the strain at which a pre-existing crack propagated by an order of magnitude. The strain at which the crack grew in the hybrid system was about twice that at which the matrix crack grew in either of the two components of the hybrid system when loaded separately. In addition, for the particular hybrid system studied, the transverse matrix crack propagated in a stable manner. This stabilizing feature was sufficient to control the growth of the crack despite the stress concentrating effects which would be expected as the crack length became comparable with the width of the specimen. In a practical system containing a distribution of matrix flaws it might be expected that stable matrix crack growth could occur sequentially from flaws of increasing severity as the tensile load applied to the composite system was increased.

Besides strengthening and stiffening the composite by carrying a tensile load the large primary reinforcing members perform two further functions. The first is to limit the elastic relaxation of the matrix and hence the amount of strain energy released by a matrix crack. Other being equal this effect is enhanced the higher the Young's modulus of the reinforcing members. However, the more effective this process becomes the more the separation of the crack faces is reduced and this will influence the energy absorbing process carried out by the second toughening fibre phase as described above.

The relaxation of the matrix causes energy to be absorbed frictionally via the debonded interface between it and the primary reinforcing members. It would appear that the matrix crack is stable because this process is not reaching its equilibrium state, as defined by the analysis set out in Section 2, for any particular crack length. In order to improve this desirable feature it may thus prove necessary to enhance the magnitude of the frictional energy loss term associated with the primary members by reducing their elastic modulus and hence their effectiveness in limiting the extent of the strain energy locking mechanism. This again, as mentioned above, has to be considered in the light of the contribution of the second reinforcing phase to the matrix work of fracture.

One important technological feature which emerges from this preliminary study is that relatively massive fibres can be used as the primary strengthening component while other smaller fibres are used as the toughening ingredient. The large fibres used here were steel wires and there is a clear economic advatage to be gained in using this material in the form of thick fibres. However, bundles of medium stiffness fibres such as glass fibres may prove to be satisfactory. Also it seems that the second toughening fibre phase could, in principle, have a low elastic modulus since it does not have to make a contribution to the macroscopic strength or stiffness of the composite. As discussed in Section 3, the energy absorbed by the second toughening fibre phase will depend on a number of factors which are influenced (through the amount by which the matrix crack faces separate) by the properties of the primary reinforcing fibres. The two fibre phases operate co-operatively in suppressing matrix crack growth and in absorbing energy during the stable growth of a matrix crack.

## 7. Conclusions

The preliminary observations reported here indicate that there could be major advantages to be gained through the design of composite systems containing fibres of very different sizes which perform distinctly different functions. As a consequence of the difference in scale, the action of the second (toughening) fibre phase can be confined to the immediate vicinity of the matrix fracture faces while the much larger primary reinforcing members exert their influence on the matrix fracture process over comparatively large distances from the fracture faces. Hence the two types of reinforcing fibres can function relatively independently but co-operatively.

Relatively small volume fractions of the two types of reinforcing fibres have been used in a very brittle matrix containing an initial severe crack. In comparison with the properties of the separate fibre-reinforced components the hybrid system carried very considerably enhanced tensile strains before the pre-existing matrix crack started to grow. Its growth proved to be stable and the results are explainable on the basis of the analysis of the fracture process given here.

Thus there are indications that practical brittle matrix hybrid composite systems can be produced which incorporate small volume fractions of reinforcing fibres and in which matrix crack growth is both suppressed and stabilized.

## Acknowledgements

The work described here was supported by the Science Research Council and the Wolfson Institute of Interfacial Technology, University of Nottingham. We thank Mr A. Martin for invaluable assistance in fabricating and testing the experimental samples.

## References

1. G. A. COOPER and J. M. SILLWOOD, *J. Mater. Sci.* 7 (1972) 325.
2. D. K. HALE and A. KELLY, "Annual Review of Materials Science", Vol. 2, edited by Robert A. Huggins (Annual Reviews Inc, Palo Alto, California, 1972) p. 405.
3. M. A. ALI and F. J. GRIMER, *J. Mater. Sci.* **4** (1969) 389.
4. A. J. MAJUMDAR, *Proc. Roy. Soc. A* **319** (1970) 69.
5. J. A. AVESTON, G. A. COOPER and A. KELLY, "The Properties of Fibre Composites" (Conf. Proc. IPC Sci. Technical Press, (1971) p. 15–26.
6. J. G. MORLEY and I. R. McCOLL, *J. Phys. D: Appl. Phys.* 8 (1975) 15.
7. I. R. McCOLL and J. G. MORLEY, *Phil. Trans. Roy. Soc.* (in press).
8. F. J. McGARRY, *Proc. Roy. Soc. A* **319** (1970) 59.
9. I. R. McCOLL and J. G. MORLEY, unpublished work.

Received 24 September and accepted 14 October 1976.